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Polynomials with One and Many Variables

Yuldoshev Mansur Najmiddin ugli

Academic Lyceum of Tashkent State University of Economics lead math science teacher

mansuryuldoshev212901@mail.ru

Abstract. In this article, polynomials, which are one of the mathematical properties, are explained in detail and explained with the help of examples and exercises.

Key words: mathematical property, polynomial, examples, coefficient, etc.

A polynomial is a mathematical expression involving a sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable (i.e., a univariate polynomial) with constant coefficients is given by

a_nx^n+...+a_2x^2+a_1x+a_0.

The individual summands with the coefficients (usually) included are called monomials (Becker and Weispfenning 1993, p. 191), whereas the products of the form x_1^(a_1)...x_n^(a_n) in the multivariate case, i.e., with the coefficients omitted, are called terms (Becker and Weispfenning 1993, p. 188). However, the term "monomial" is sometimes also used to mean polynomial summands without their coefficients, and in some older works, the definitions of monomial and term are reversed. Care is therefore needed in attempting to distinguish these conflicting usages. The highest power in a univariate polynomial is called its order, or sometimes its degree. Any polynomial P(x) with P(0)!=0 can be expressed as

$$P(x) = P(0)\prod_{\rho}\left(1 - \frac{x}{\rho}\right),$$

where the product runs over the roots rho of P(rho)=0 and it is understood that multiple roots are counted with multiplicity. A polynomial in two variables (i.e., a bivariate polynomial) with constant coefficients is given by

$$a_{nm}\,x^n\,y^m + \ldots + a_{22}\,x^2\,y^2 + a_{21}\,x^2\,y + a_{12}\,x\,y^2 + a_{11}\,x\,y + a_{10}\,x + a_{01}\,y + a_{00}.$$

Similarly, a polynomial of fifth degree may be computed with four multiplications and five additions, and a polynomial of sixth degree may be computed with four multiplications and seven additions. Polynomials of orders one to four are solvable using only rational operations and finite root extractions. A first-order equation is trivially solvable. A second-order equation is soluble using the quadratic equation. A third-order equation is solvable using the cubic equation. A fourth-order equation is solvable using the quartic equation. It was proved by Abel and Galois using group theory that general equations of fifth and higher order cannot be solved rationally with finite root extractions (Abel's impossibility theorem). However, solutions of the general quintic equation may be given in terms of Jacobi theta functions or hypergeometric functions in one variable. Hermite and Kronecker proved that higher order polynomials are not soluble in the same manner. Klein showed that the work of Hermite was implicit in the group properties of the icosahedron. Klein's method of solving the quintic in terms of hypergeometric functions in one variable can be extended to the sextic, but for higher order polynomials, either hypergeometric functions in several variables or "Siegel functions" must be used (Belardinelli 1960, King 1996, Chow 1999). In the 1880s, Poincaré created functions which give the solution to the nth order polynomial equation in finite form. These functions turned out to be "natural" generalizations of the elliptic functions.

Many common functions are polynomial functions. In this unit we describe polynomial functions and look at some of their properties. In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature. After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- recognise when a rule describes a polynomial function, and write down the degree of the polynomial,
- recognize the typical shapes of the graphs of polynomials, of degree up to 4,
- understand what is meant by the multiplicity of a root of a polynomial,
- sketch the graph of a polynomial, given its expression as a product of linear factors.

You may recall that when (x − a)(x − b) = 0, we know that a and b are roots of the function f(x) = (x − a)(x − b). Now we can use the converse of this, and say that if a and b are roots, then the polynomial function with these roots must be f(x) = (x − a)(x − b), or a multiple of this. For example, if a quadratic has roots x = 3 and x = −2, then the function must be f(x) = (x−3)(x+2), or a constant multiple of this. This can be extended to polynomials of any degree. For example, if the roots of a polynomial are x = 1, x = 2, x = 3, x = 4, then the function must be f(x) = (x − 1)(x − 2)(x − 3)(x − 4), or a constant multiple of this. Let us also think about the function f(x) = (x − 2)2 . We can see straight away that x − 2 = 0, so that x = 2. For this function we have only one root. This is what we call a repeated root, and a root can be repeated any number of times. For example, f(x) = (x − 2)3 (x + 4)4 has a repeated root x = 2, and another repeated root x = −4. We say that the root x = 2 has multiplicity 3, and that the root x = −4 has multiplicity 4. The useful thing about knowing the multiplicity of a root is that it helps us with sketching the graph of the function. If the multiplicity of a root is odd then the graph cuts through the x-axis at the point (x, 0). But if the multiplicity is even then the graph just touches the x-axis at the point (x, 0). For example, take the function

$$f(x) = (x-3)^2(x+1)^5(x-2)^3(x+2)^4.$$

- The root x = 3 has multiplicity 2, so the graph touches the x-axis at (3, 0).
- The root x = −1 has multiplicity 5, so the graph crosses the x-axis at (−1, 0).
- The root x = 2 has multiplicity 3, so the graph crosses the x-axis at (2, 0).
- The root x = −2 has multiplicity 4, so the graph touches the x-axis at (−2, 0).

To take another example, suppose we have the function f(x) = (x − 2)2 (x + 1). We can see that the largest power of x is 3, and so the function is a cubic. We know the possible general shapes of a cubic, and as the coefficient of x 3 is positive the curve must generally increase to the right and decrease to the left. We can also see that the roots of the function are x = 2 and x = −1. The root x = 2 has even multiplicity and so the curve just touches the x-axis here, whilst x = −1 has odd multiplicity and so here the curve crosses the x-axis. This means we can sketch the graph as follows. Now, consider a square of side 3 units. What is its perimeter? You know that the perimeter of a square is the sum of the lengths of its four sides. Here, each side is 3 units. So, its perimeter is 4 × 3, i.e., 12 units. What will be the perimeter if each side of the square is 10 units? The perimeter is 4 × 10, i.e., 40 units. In case the length of each side is x units (see Fig. 2.2), the perimeter is given by 4x units. So, as the length of the side varies, the perimeter varies.

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