

Free Vibrations of a Plate Stretched along the Reinforced SidesN. Melikulov¹, N. U. Shodmonkulova²¹Ph.D., Associate Professor Sam SASI, "Building Mechanics and Strength of Materials"²assistant Sam SASI, "Building mechanics and strength of materials"

ABSTRACT: To study the effect of tensile forces on the frequencies of free vibrations, consider a rectangular plate, two parallel edges of which are hinged, and the other two are supported by thin-walled ribs of an open profile.

KEYWORDS: Compressed, stretched, free vibrations, frequency, elastic pinching, hinged, torsional, bending, reinforced, non-reinforced, edges, stiffness.

Problem statement and literature review. As far as we know, the free vibrations of the reinforced plates, taking into account the torsion of the reinforcing ribs, are still insufficiently studied. This is especially true for plates supported by thin-walled open profile rods. Taking into account the specificity of constrained torsion of thin-walled ribs, as will be shown below, significantly affects the stiffness parameters of the "plate-rib" system and, as a consequence, the frequencies of its natural vibrations.

The literature [2, 3, 4, 6] describes the following method of drawing up the boundary conditions along the line of contact of the plate with the reinforcing ribs. The loads transferred by the plate to the ribs (rods) are considered equal, but opposite in direction to the forces in the corresponding sections of the plate. Then, the kinematic conditions of equality of displacements at the points of contact of the reinforcing rods with the plate are introduced into these force conditions.

In publications [1, 5], a technique is proposed for drawing up refined boundary conditions on the line of conjugation of the plate with the rod, which makes it possible to take into account the constraint in the warping of the end sections of the ribs. In this case, the degree of constraint of warping is taken into account by some parameter. This parameter is included in the expression for the generalized coefficient of elastic pinching of the edges of the plate and depends not only on the geometric and mechanical characteristics of the bar and the plate, but also on the force transmitted to the edge and on the shape of the buckling of the plate.

Solution method. As it is known, a closed solution to the problem of free vibrations of a rectangular plate can be obtained in single trigonometric series only when two parallel edges are hinged, while the other two edges can be fixed in an arbitrary way (M. Levy's solution).

To study the effect of tensile forces on the frequencies of free vibrations, consider a rectangular plate, two parallel edges of which are hinged, and the other two are supported by thin-walled ribs of an open profile. Longitudinal forces of intensity N_x are applied to the unsupported hinged edges (at the level of the median plane) and to the ends of the reinforcing ribs (Fig. 1).

Let us write down the well-known differential equation of motion of the curved surface of a compressed plate for small deflectionsthe solution of which is presented in the traditional form,

$$D \nabla^2 \nabla^2 w - N_x \frac{\partial w}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} = 0,$$

$$w(x,y,t) = \sum_{n=1,2,\dots}^{\infty} f_n(y) \sin \lambda_n x \cos \omega t,$$

satisfying the boundary conditions of hinge support at the edges $x = 0, a$.

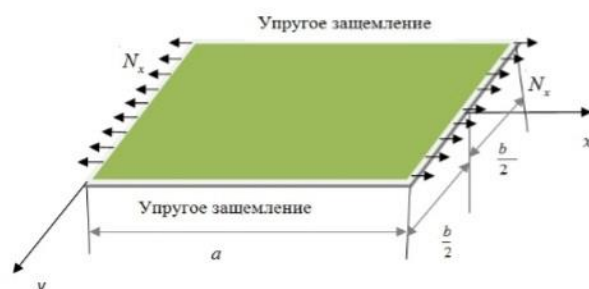


Fig 1. Plate, two parallel edges of which are hinged, and the other two are supported by thin-walled ribs of an open profile.

Substituting (2) into (1), we obtain the equation for determining the desired function $f = f_n(y)$:

$$f_n^{IV} - 2\lambda_n^2 f_n'' + \left[\lambda_n^2 \left(\lambda_n^2 + \frac{N_x}{D} \right) - \omega_*^2 \right] f_n = 0, \quad \omega_* = \omega \sqrt{\frac{\rho h}{D}}.$$

where,

We represent the solution of equation (3) in the form

$$\alpha_n = \sqrt{\lambda_n^2 + \sqrt{\omega_*^2 + \lambda_n^2 N_x / D}}, \quad \beta_n = \sqrt{\omega_*^2 + \lambda_n^2 N_x / D - \lambda_n^2}$$

To concretize further calculations, we introduce a particular simplifying assumption that the ribs reinforcing the plate and the conditions for fixing their ends are the same. This assumption makes it possible to use the symmetry of the curvature of the plate and to accept in equation (4) the constants $C_2 = C_4 = 0$ for symmetric and $C_1 = C_3 = 0$ for skew-symmetric vibrations with respect to axial modes.

Thus, the conditions of elastic pinching at $y = \pm b / 2$ are assumed to be the same.

We now subject the function $f_n(y)$ to the boundary conditions

$$f_n' = (\mu \lambda_n^2 f_n - f_n'') / t_k \lambda_n^2 b; \quad f_n \left(\pm \frac{b}{2} \right) = 0$$

The absence of a trivial solution $f_n(y) = 0$ in the resulting system of homogeneous equations leads to the equality of the corresponding determinant to zero. Its expansion leads to the following particular equation: for symmetrical vibration modes

$$(\eta \sin \eta + \xi \operatorname{th} \xi \cos \eta) t_k \lambda_n^2 b + 2(\xi^2 + \eta^2) \cos \eta = 0,$$

for skew-symmetric shapes

$$(\xi \sin \eta - \eta \operatorname{th} \xi \cos \eta) t_k \lambda_n^2 b - 2(\xi^2 + \eta^2) \operatorname{th} \xi \sin \eta = 0$$

The values of the parameters ξ , η , and t_k are given

$$\xi = \frac{\alpha_n b}{2}, \eta = \frac{\beta_n b}{2}$$

$$t_k = \frac{k_3^2 C_k}{k^2 d_k} \left(\frac{\lambda_n^2}{k_3^2} + 1 \right), C_k = \frac{GJ_k}{Db}$$

The relationship between ξ and η and also the expression for the vibration frequency follows from (5):

$$\xi^2 - \eta^2 = \frac{1}{2} \lambda_n^2 b^2, \omega_* b^2 = 2 \sqrt{(\xi^2 + \eta^2)^2 + \pi^2 \lambda_n^2 b^2 \frac{N_x}{N_\vartheta}}, N_\vartheta = \frac{4\pi^2 D}{b^2}$$

where N_ϑ - the critical value of the intensity for a square plate hinged at all edges.

Let us further concretize the coefficient t_k taking into account the influence of longitudinal forces transmitted to the reinforcing ribs. We will assume that the "plate - ribs" system is in a uniform stress state, that is,

$$\sigma_x = \frac{N_x}{h} = \frac{P}{F},$$

where h is the thickness of the plate

F is the cross-sectional area of thin-walled ribs.

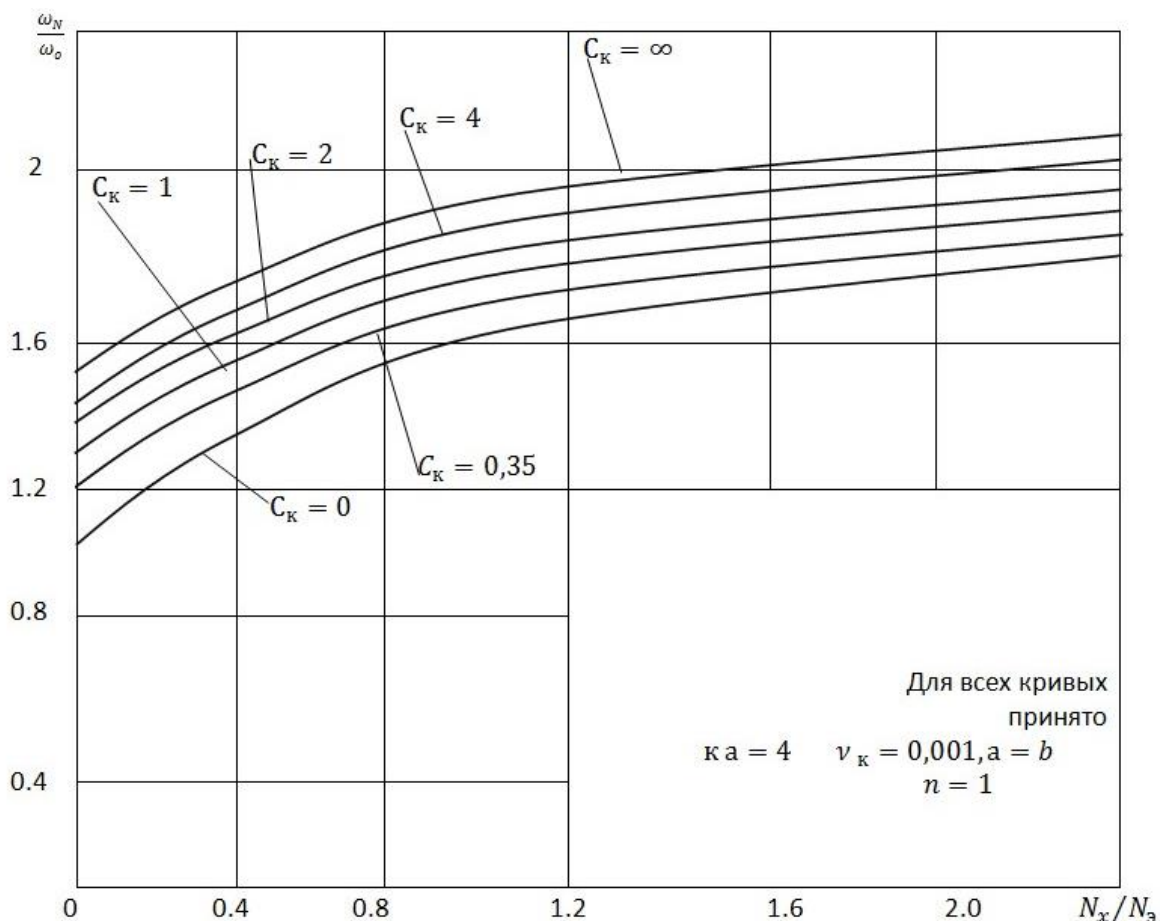
$$P = \frac{N_x F}{h} \quad k_2^2 = \frac{Pr^2}{EJ_\omega} + k^2,$$

Substituting the value into expression

$$k^2 = \frac{GJ_k}{EJ_\omega} \quad \text{--- and then the last - in (10) we get}$$

$$t_k = \left[\left(\frac{n\pi}{ka} \right)^2 + \frac{4V_k \pi^2}{C_k} \cdot \frac{N_x}{N_3} + 1 \right] \frac{C_k}{d_k}$$

Here $V_k = J_0 / hb^3$ is a coefficient that determines the share of the total load transferred to each of the ribs. J_0 is the polar moment of inertia of the rib section, calculated relative to the point of contact of the rib with the plate.



2 Fig. Graph of the dependence of the vibration frequency of a square plate on the tension intensity

To illustrate the wobbling of forces N_x acting at the level of the median plane of the plate along the reinforced edges, we present some results of numerical analysis (Fig. 2).

To plot the graphs, transcendental equations (8) and (9) were used, which were solved together with expressions (11) and (13).

Note that the lower curve corresponds to the case of the plate hinged along the entire contour; the upper curve is plotted for a plate rigidly clamped at the edges $y = \pm b/2$.

A natural conclusion follows from the graph that the frequency decreases with increasing plate stretching and that it vanishes when N_x reaches the critical value. With an increase in the parameter C_k , which characterizes the torsional stiffness of the ribs, the frequency of free vibrations increases significantly. This regularity, which is quite natural (because with an increase in C_k , the total bending stiffness of the plate increases), can be traced both in the case of an unloaded plate ($N_x = 0$) and under the action of forces N_x of various intensities. Note that an increase in the intensity of tensile forces is accompanied by a noticeable increase in the frequency of free vibrations.

Conclusion. Used in solving problems of free vibrations of reinforced plates, dimensionless coefficients reflecting the degree of elastic support and elastic pinching of the edges, take into account the mechanical and geometric characteristics of the plate and ribs, constructive methods of restraining deformation and attaching ribs to the plate, as well as the influence of longitudinal forces perceived by the ribs. An essential feature of these coefficients is their dependence on the number of half-waves of the plate curvature formed in the process of oscillations.

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