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Methods of Solving Some Problems from Selected Chapters of Geometry

Rasulova Gulnozakhan Azamovna, Ph.D

Senior teacher, Doctor of Philosophy in Pedagogical Sciences, Kokand State Pedagogical Institute, Department of Mathematics

Abstract: Methodological recommendations for solving typical problems from selected chapters of geometry are given in the article. It describes the formation of skills of students to correctly identify geometric relationships and know the results based on resourcefulness by providing methods of solving some problems.

Keywords: typical problems, algebraic substitution, bisector, Menelaus and Stewart's theorems.

Since the scope of geometry is a broad science, we have to choose our own way of solving geometric problems, depending on the given problem and based on its characteristics.

In general, it depends on familiarity with typical methods of problem solving, the ability to find relationships of various mathematical sentences, and ingenuity in seeing some unexpected results arising from some theorems and their combinations.

It consists of proving the sentences related to the solution of the problem and making the main and auxiliary forms related to it, determining their mutual relations, and then performing the necessary algebraic substitution and calculations.

Below are examples of such typical issues.

Issue 1. Parties a,b,c find the length of the bisector of the triangle

Solution: ABC in a triangle $AA'=l_a$ let it be

$BA':A'C=c:b$ Taking into account, BA' and $A'C$ we find the length of the sections.

$BA'+A'C=a$

if we take into account that, the following result is obtained:

$$BA'=\frac{ac}{b+c},\ A'C=\frac{ab}{c+c}.$$

Then according to Stewart's theorem:

$$l_a^{\,2}\cdot a=\frac{ab^2c}{b+c}+\frac{abc^2}{b+c}-\frac{ac}{b+c}\cdot\frac{ab}{b+c}\cdot a\,.$$

After such simple substitutions

$$l_a^{\,2}=bc\cdot\frac{(b+c)^2-a^2}{(b+c)^2}$$

originates. Here $a+b+c=2p,\ \ b+c-a=2(p-a)$ assuming that:

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$$l_a^{\,2} = \frac{4bcp(p-a)}{(b+c)^2} \quad \text{yoki} \quad l_a^{\,2} = \frac{2}{b+c}\sqrt{bcp(p-a)}.$$

Issue 2. A circle inscribed outside the triangle (radius R)) triangle with center (sides a,b,c) find the distance between the center of gravity.

Solution: ABC triangle BC the midpoint of the side D and the distance sought

OG let it be. OBD from the triangle: $OD^2 = R^2 - \frac{a^2}{4}$ and $AD = m_a$, Stewart applying the theorem:

$$OG^2 \cdot AD = OA^2 \cdot DG + OD^2 \cdot AG - AD \cdot AG \cdot DG$$

or

$$OG \ \cdot m = R^2 \cdot \frac{1}{3}m_a + (R^2 - \frac{a^2}{4}) \cdot \frac{2}{3}m_a - m_a \cdot \frac{1}{3}m_a \cdot \frac{2}{3}m_a,$$

ago:

$$OG^2 = \frac{1}{3}R^2 + \frac{2}{3}(R^2 - \frac{a^2}{4}) - \frac{2}{9}m^2{}_a = R^2 - \frac{a^2+b^2+c^2}{9},$$

so:

$$OG = \frac{1}{3}\sqrt{9R^2 - (a^2+b^2+c^2)}.$$

Issue 3. One straight line ABC of the triangle AB, AC sides and BC the continuation of the side respectively A', B', C' intersects at points. BC in relation to the middle of the side A' symmetrical A'' and CA in relation to the middle of the side B' symmetrical to a point B'' and AB relative to the middle of the side C'' points earned. That's it A'', B'', C'' prove that the points are collinear, i.e. they lie on a straight line.

Proof. According to Menelaus' theorem:

$$\frac{BA'}{A'C} \cdot \frac{CB'}{B'A} \cdot \frac{AC'}{C'A} = 1.$$

Besides:

$$BA'' = A'C, \ A''C = BA', \ CB'' = B'A,$$

$$B''A = CB', \ AC'' = C'B, \ C''B = AC'.$$

That is why:

$$\frac{BA''}{A''C} \cdot \frac{CB''}{B''A} \cdot \frac{AC''}{C''B} = 1$$

from this last equality according to Menelaus' theorem A'', B'', C'' it follows that the points lie on a straight line.

Issue 4. ABC of the triangle CA, CB the radius of the circle trying to the sides and the circle drawn outside it $r*$ for

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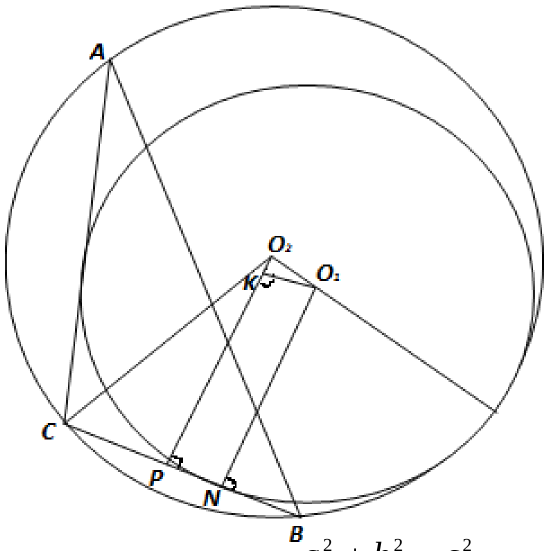
$$r^* = \frac{ab}{p}\sqrt{\frac{(p-a)(p-b)}{p(p-c)}} \qquad (1)$$

prove that the equality holds. In this $a = |BC|, b = |AC|, c = |AB|$ being, p -semi-perimeter of the triangle.

Proof. ABC a triangle with obtuse angles $(\angle C > 90°)$ we prove that. ABC the center of the circle inscribed outside the triangle O_2, CA, CB and the center of the circle trying to circumscribe the triangle O_1 let it be. This is a circle AC towards N try at the point. $O_2P \perp CA$, $O_1K \perp O_2P$ we will make cuts.

$\angle A = \alpha$, $\angle C = \gamma$ let it be. In that case $tg\frac{\gamma}{2} = \frac{O_1N}{CN} = \frac{r^*}{CN}$.



Drawing 1

Ago $$CN = r^* ctg\frac{\gamma}{2} = r^*\frac{1+\cos\gamma}{\sin\gamma} = r^*\frac{1+\frac{a^2+b^2-c^2}{2ab}}{\sin\gamma} = r^*\frac{(a+b-c)(a+b+c)}{2ab\sin\gamma} =$$

$$= r^*\frac{4(p-c)p}{4S_{\Delta ABC}} = r^*\sqrt{\frac{p(p-c)}{(p-a)(p-b)}}.$$

O_1KO_2 in a triangle $O_1O_2 = R - r^*$, $O_1K = CN - CP = r^*\sqrt{\frac{p(p-c)}{(p-a)(p-b)}} - \frac{a}{2}$,

$O_2K = O_2P - r^* = \sqrt{R^2 - \left(\frac{a}{2}\right)^2} - r^*$. Here R-is the radius of the circle inscribed outside the triangle. Now ΔO_1KO_2 we use the Pythagorean theorem for. $O_1K^2 + O_2K^2 = O_1O_2^2$,

$$\left(\sqrt{R^2 - \left(\frac{a}{2}\right)^2} - r^*\right)^2 + \left(r^*\sqrt{\frac{p(p-c)}{(p-a)(p-b)}} - \frac{a}{2}\right)^2 = \left(R^2 - r^*\right)^2. \qquad (2)$$

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Now let's do some shape swapping:

1. $$\sqrt{R^2-\left(\frac{a}{2}\right)^2}=\sqrt{R^2-R^2\sin^2\alpha}=R|\cos\alpha|=R\cos\alpha=R\frac{b^2+c^2-a^2}{2bc}=$$
$$=R\left(\frac{(b-c)^2-a^2}{2bc}+1\right)=R\left(1-\frac{2(p-b)(p-c)}{bc}\right).$$

2. $$\frac{4R(p-b)(p-c)}{bc}=\frac{abc}{S_{\Delta ABC}}\cdot\frac{(p-b)(p-c)}{bc}=a\cdot\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}.$$

(2) from

$$-2\sqrt{R^2-\frac{a^2}{4}}+r*\frac{p(p-c)}{(p-a)(p-b)}-a\sqrt{\frac{p(p-c)}{(p-a)(p-b)}}=-2R$$

equality is formed. From this

$$-2R\left(1-\frac{2(p-b)(p-c)}{bc}\right)+r*\frac{p(p-c)}{(p-b)(p-a)}=-2R,$$
$$\frac{4R(p-b)(p-c)}{bc}+r*\frac{p(p-c)}{(p-a)(p-b)}-a\sqrt{\frac{p(p-c)}{(p-a)(p-b)}}=0,$$
$$a\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}+r*\frac{p(p-c)}{(p-a)(p-b)}-a\sqrt{\frac{p(p-c)}{(p-a)(p-b)}}=0,$$
$$r*\frac{p(p-c)}{(p-a)(p-b)}=a\sqrt{\frac{p-c}{p-a}}\left(\sqrt{\frac{p}{p-b}}-\sqrt{\frac{p-b}{p}}\right),$$
$$r*=\frac{ab}{p}\sqrt{\frac{(p-a)(p-b)}{p(p-c)}}.$$

So, equality (1) holds.

Issue 5. So that α,β,γ if there are interior angles of an arbitrary triangle, then prove the $\sin^2\alpha+\sin^2\beta+\sin^2\gamma\le\frac{9}{4}$ following inequality.

Solution: Exiting the plane from one point $\overline{r_1},\overline{r_2},\overline{r_3}$ we get unit vectors. The angles between them $2\alpha,2\beta,2\gamma$ let it be. Then this $(\overline{r_1},\overline{r_2},\overline{r_3})^2\ge 0$ according to inequality

$$3+2\cos 2\alpha+2\cos 2\beta+2\cos 2\gamma\ge 0,$$
$$3+2\left[3-2\sin^2\alpha-2\sin^2\beta-2\sin^2\gamma\right]\ge 0.$$

From the last inequality $\sin^2\alpha+\sin^2\beta+\sin^2\gamma\le\frac{9}{4}$ originates.

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Exercises for independent solving

1. If the angles A, B end C in triangle ABC are in the ratio 4:2:1 then for its sides $\frac{1}{a}+\frac{1}{b}+=\frac{1}{c}$ prove that the equality holds.
2. Two circles with radius R end r are external to each other and have a common radius. Find the radius of the circle inscribed in the resulting curved triangle.
3. Sides of triangle ABC $a^2+b^2=5c^2$ Prove that its two medians are perpendicular if connected by a relation.
4. So that $S-ABC$ of the triangle, S_1-ABCD if a parallelogram is a face, prove the following equalities:

   a) $S=\frac{1}{2}\sqrt{\overline{AB}^2\cdot\overline{AC}^2-(\overline{AB}\cdot\overline{AC})^2}$;

   b) $S_1=\sqrt{\overline{AB}^2\cdot\overline{AD}^2-(\overline{AB}\cdot\overline{AD})^2}$.
5. An equilateral triangle is made with the diameter of the semicircle 2r find the face of the triangle that remains outside the circle.
6. Center A,B,C at points and radii respectively a,b,c is the largest to the shape bounded by these circles when the circles intersect from the outside r. Prove that the following formula holds if the circle with radius is inscribed. $r=\frac{abc}{ab+bc+ac+2\sqrt{abc(a+b+c)}}$
7. The four sides of a rectangle inscribed in a circle a,b,c,d find his face by.
8. For any triangle ABC

   a) $\cos\hat{A}+\cos\hat{B}+\cos\hat{C}\le\frac{3}{2}$;

   b) $\cos 2\hat{A}+\cos 2\hat{B}-\cos 2\hat{C}\le\frac{3}{2}$
9. A circle is drawn from one end of the square with a radius equal to half of the side of the square, and an attempt is made to circle from the adjacent end. How will this attempt divide the face of the square?
10. Of the triangle h_1,h_2,h_3 derive the formula of its face in terms of heights.
11. Triangulate from a point inside the triangle a,b,c to the sides p_1,p_2,p_3 perpendiculars are passed and their bases are connected. Find the face of the triangle formed by connecting the bases of the perpendiculars.
12. Radius r the circle is surrounded by three equal circles. They try to each other and to the given circle. Find the radius of one of these circles.

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