

Property of the Cowell Method for Solving Differential Equations**Eshmamatova Dilfuza Baxramovna**

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Abstract

Differential equations are the basis for the study of continuous physical processes of mechanical systems because it is one of the mathematical apparatus dynamics, stagnation theory and study of physical concepts such as vibration theory, in most cases leading to the solution of differential equations or the application of the theory of quality.

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Alternatively, the course of differential equations of higher mathematics is an integral part of all departments, it is also used as a basic and necessary apparatus for postgraduate and parallel mathematics and some physics courses.

Many processes that occur in physics, economics, biology, chemistry, medicine and other sciences are described using differential equations. By studying the differential equations, we gain information about the relevant processes. These differential equations are a mathematical model of the process under study, the study of differential equations leads to a complete description of the processes.

Many natural phenomena in the macro-and micro world are described by the same differential equations. Moreover, in most cases, the general solution of these equations is not expressed in quadratures, i.e., it does not have an analytical solution. For this class of problems, it becomes necessary to use numerical methods that give an approximate solution to the problem.

Most of the numerical methods for integrating differential equations can be divided into two groups based on two different principles.

The methods of the first group are based on the Taylor series expansion, and the derivatives of higher orders are determined by differentiating the right-hand side of this equation analytically or numerically. This group includes the methods of Euler, Adams, Stermer, Cowell, etc.

The methods of the second group are also based on series expansion, but without using in the process of calculating higher-order derivatives. Instead, at this step, the right-hand side of the equation is calculated for various values of the variables included there. These methods are called Runge-Kutta methods.

For methods of the second group, unlike the methods of the first group, no other methods are required to start the integration, since to move forward one step, only the values of the sought functions at the last point are used. In addition, the step size can be changed at any time during the

integration process. In this case, the results of miscalculations with normal and half steps can serve as a criterion for changing the step [1].

However, these methods are not free of disadvantages, since the integration process is devoid of simple means of controlling the correct selection of the step and detecting arithmetic errors. The method requires a lot of calculations of the right-hand sides of the differential equation, which is associated with a lot of computational work, especially when the right-hand sides of differential equations have a complex structure. Increasing the accuracy of the method is associated with great difficulties, since this increases the complexity of the working formulas.

However, when using a computer, these disadvantages are not of fundamental importance.

The methods of the first group are devoid of the indicated disadvantages; therefore they have found wide application for solving problems, requiring a high degree of accuracy. The most common is the Cowell method, which is identical to the Gaussian “mechanical quadrature” method. In particular, Cowell’s method is most often used in solving problems of celestial mechanics [2].

Consider a system of differential equations of the following form:

$$\frac{d^2 r}{dt^2} = R(r, t) \quad (1)$$

A feature of equation (1) is that, in its right-hand side the first derivative is absent and therefore two integrations can be avoided-however, to obtain r , and the other to find r . The basic idea behind Cowell’s method is, to directly calculate the required function r of the second derivative r , by the double integration formula, using the Stirling interpolation formula. Let us present the derivation of the basic relations of Cowell’s method up to terms of the twelfth order inclusive with respect to the integration step h .

According to Taylor’s formula:

$$r(t_k - h) = r_{k-1} = r_k + \sum_{n=1}^{\infty} (-1)^n \frac{h^n}{n!} \left(\frac{d^n r}{dt^n} \right)_k$$

$$r(t_k + h) = r_{k+1} = r_k + \sum_{n=1}^{\infty} (-1)^n \frac{h^n}{n!} \left(\frac{d^n r}{dt^n} \right)_k$$

From these relations we find the second differences

$$\Delta^2 r_k = 2 \sum_{n=1}^{\infty} \frac{h^{2n}}{(2n)!} \left(\frac{d^{2n} r}{dt^{2n}} \right)_k$$

Since

$$\Delta^2 r_k = r(t_k + h) - 2r(t_k) + r(t_k - h)$$

Assuming

$$f = h^2 r = h^2 R(r, t)$$

We obtain

$$\Delta^2 r_k = f_k + 2 \sum_{n=1}^{\infty} \frac{h^{2n}}{(2n+2)!} \left(\frac{d^{2n} r}{dt^{2n}} \right)_k$$

Using Stirling's interpolation formula, we express the derivatives in terms of the difference of the function, using for this purpose.

When using these formulas to integrate differential equations, it is necessary to match the accuracy of these formulas with the specified degree of accuracy of the final result. In this regard, it becomes necessary to estimate the accumulation of errors in numerical integration by the Cowell method. The principal disadvantage of this method is the rapid accumulation of round-off errors. After n steps of numerical integration are done, errors in the obtained solution turn out to be proportional to $\frac{3}{n^2}$ [3,4].

Conclusion

Having considered various difference methods for solving ordinary differential equations, both universal (methods of Taylor expansions, Runge-Kutte, Adams-Bashforth, Adams-Multon) and specialized (Everhart, Cowell), the authors came to the following conclusions.

Comparison of universal methods can be carried out on the basis of several principles: a) precision; b) reliability; c) direct costs and d) full costs.

The specialized Cowell method can be effectively used to solve the equations of motion in problems of celestial mechanics.

Literature

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