

Calculating the Total Resistance of Some Schemes by Using Kirchhoff's Laws

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Annotation: This paper presents methods for calculating the resistance of different circuits using Kirchhoff's laws.

Key words: node, entering current, leaving current, contour, D.F (driving force)

In the "Electricity and Magnetism" section of the General Physics course, some difficulties arise in solving problems related to Kirchhoff's laws, especially when calculating the total resistance of some circuits. In this article, using Kirchhoff's laws, especially when calculating the total resistance of some schemes are given. The emphasis is directed for the laws of nodes and the symmetry of drawings.

Before solving the problems, let's take a brief look at Kirchhoff's laws.

Kirchhoff's first law: The algebraic sum of the currents entering the node is equal to the algebraic sum of the currents leaving the node.

$$\sum I_k = \sum I_{ch}. \quad (1)$$

Here I_k - are the currents entering the node, and I_{ch} - are the currents leaving the node.

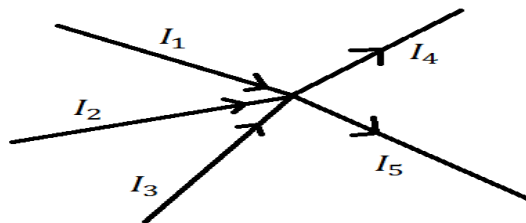


Figure 1. Drawing of Kirchhoff's first law

Using figure 1, we can write Kirchhoff's first law as follows:

$$I_1 + I_2 + I_3 = I_4 + I_5. \quad (2)$$

Kirchhoff's second law: The algebraic sum of the voltage drops across all resistances (including the internal resistance of the source) as it rotates around any closed circuit in a complex electrical circuit is equal to the algebraic sum of the driving forces of that circuit (D.F)

$$\sum_i I_i R_i = \sum_j \mathcal{E}_j. \quad (3)$$

The direction of rotation of each contour (clockwise or counterclockwise) is optional. If the pre-selected current direction in the section between the two nodes coincides with the direction of rotation of the circuit, then the voltage drop is positive, if the current direction is opposite to the direction of rotation, the voltage is negative. If the current source is thrown from the negative pole to the positive pole while rotating along the contour, then D.F is considered positive, otherwise D.F is considered negative.

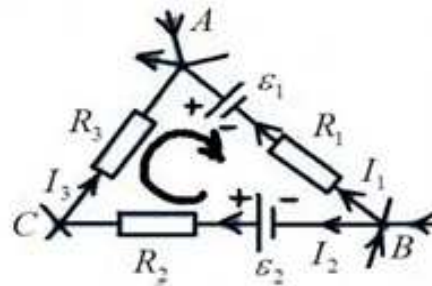


Figure 2. Drawing of Kirchhoff's second law.

Using figure 2, we can write Kirchhoff's second law as follows:

$$-I_1R_1 - I_1r_1 + I_2R_2 + I_2r_2 + I_3R_3 = -\varepsilon_1 + \varepsilon_2 . \quad (4)$$

Above we have read Kirchhoff's laws. Below we consider the issues of finding the total resistance of some schemes using these laws.

Problem 1: A resistor is connected to each side of the cube. What is the resistance of a cube when it is connected to a network with ends with large diagonal ends? (figure 3)

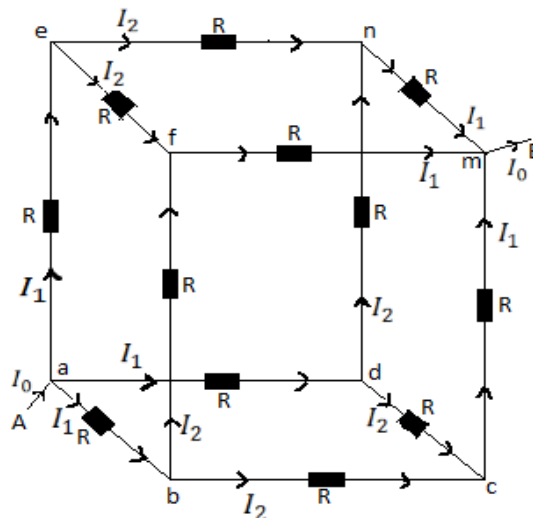


Figure 3 . Drawing of cube.

Solution: We use the laws of symmetry to solve this problem. In Figure 3, a current I_0 flows in three directions entering node **a**, the point from point **A** to point **B** is the same for all three directions are the same and equal to I_1 . Similarly, we use the laws of symmetry for nodes **b**, **d** and **e**. A current I_1 enters each of these nodes, and the path from these nodes to the node is the same, so the current from each node is the same and equal to I_2 . We now use the inverse of the currents, that is, if the currents coming from node **a** are equal to I_1 , then the currents coming from node **m** are also equal to I_1 .

Using Kirchhoff's laws, we write:

For the node **a**: $I_0 = 3I_1$.(5)

For the node **b**: $I_0 = 2I_2$.(6)

For a part of the chain, it is known from Ohm's law:

$$U_{AB} = R_u I_0 . \quad (7)$$

R_u - the fiber resistance of the chain, U_{AB} - the voltage between points A and B.

On the other hand, using Figure 3, we follow an arbitrary path from point A to point B to find the voltage between points A and B. For example, $a \rightarrow b \rightarrow c \rightarrow m$ along the path. The voltage:

$$U_{AB} = I_1R + I_2R + I_1R = 2I_1R + I_2R . \quad (8)$$

We equate (7) and (8) and find R_u using (5) and (6)

$$R_u I_0 = 2I_1R + I_2R .$$

$$R_u \cdot 6I_2 = 5I_2R .$$

Answer: $R_u = \frac{5}{6}R .$

Problem 2: A resistor is connected to each side of the cube. What is the resistance of the base of the cube when it is connected to the network with its diagonal ends (figure 4).

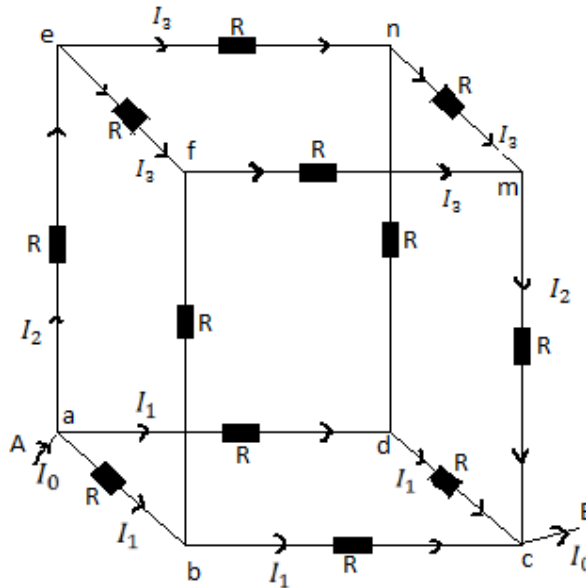


Figure 3 . Drawing of cube.

Solution: From the law of symmetry we write:

- 1) Since the directions $a \rightarrow d \rightarrow c$ and $a \rightarrow b \rightarrow c$ are the same, the currents in **ad** and **ab** are the same and equal to I_1 .
- 2) Since the direction $e \rightarrow n \rightarrow m \rightarrow c$ and $e \rightarrow f \rightarrow m \rightarrow c$ are the same, the currents in **en** and **ef** are the same and equal to I_2 .

Using the reverse direction of the currents, we write:

- 1) If the currents in parts **ad** and **ab** from node **a** are equal to I_1 , then the currents in parts **bc** and **dc**, which are symmetrical to parts **ad** and **ab** entering node **c**, are also equal to I_1 .
- 2) If the current in the **ae** part of the node **a** is equal to the current I_2 , then the current in the **mc** part, which is symmetrical in the **ae** part, is also equal to I_2 .
- 3) If the currents in parts **en** and **ef** from node **e** are equal to I_3 , the currents in parts **nm** and **fm**, which are symmetrical to parts **en** and **ef** entering node **m**, are also equal to I_3 .

As can be seen from the diagram, no current flows through the **bf** and **nd** parts. That is, we can assume that the resistances in these parts are equal to 0.

For the node **a**: $I_0 = 2I_1 + I_2 . \quad (9)$

For the node **b**: $I_2 = 2I_3 . \quad (10)$

For contour $b \rightarrow f \rightarrow m \rightarrow c$: $I_3R + I_2R - I_1R = 0$, henceforth

$$I_3 + I_2 - I_1 = 0 . \quad (11)$$

For a part of the chain, it is known from Ohm's law:

$$U_{AB} = R_u I_0 . \quad (12)$$

R_u - the fiber resistance of the chain, U_{AB} - the voltage between points A and B.

On the other hand, using Figure 3, we follow an arbitrary path from point A to point B to find the voltage between points A and B. For example, $a \rightarrow b \rightarrow c$ along the path. The voltage:

$$U_{AB} = I_1 R + I_1 R = 2I_1 R . \quad (13)$$

we equalize (12) and (13) and use (9), (10) and (11) and find R_u .

$$R_u I_0 = 2I_1 R .$$

$$R_u \cdot 8I_3 = 6I_3 R .$$

Answer: $R_u = \frac{3}{4} R .$

Using the process of solving the problems discussed above, we can say that the solution is easier to find if we also use Kirchhoff's laws to find the resistance of schemes. In this case, we construct equations using the law of nodes and the laws of contours. We are also required to set the values and directions of the currents correctly. The solution is easier to find when we use the symmetry of the circuit and the inverse of the currents to set the values and directions of the currents.

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