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Solving Problems of Minimization of Norms in a Finite Time Interval Using the Problem of Moments

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Abstract:

We consider the problem of minimizing the value of t_1 in the time interval $[t_0, t_1], t_1 - t_0 = T$ in the field D. Let t_0 be given. Let`s consider the problem of minimizing the value of T with the restriction of the control norm $u(t)$ in the field D and the value of T. In this case, the quantitiy λ_n defined by equations (1) or (2) in the previous section can be called a function of T, that is $\lambda_n = \lambda_n(T)$. Then the minimum value of $T = T^*$ is the smallest real non-negative value of T and at this value.

Keywords: minimizing, interval, non-negative.

$$\lambda_n(T) \le l \qquad (1)$$

the inequality is fulfilled, to determine T, the following equality can be obtained instead of equality (1):

$$\lambda_n(T) = l \qquad (2)$$

The sought T^* (2) is the smallest real and non-negative root of the equation. It is based on the fact thet the function $\lambda_n(T)$ is a non-increasing function of the argument T. This is turn, follows from the fact that the function $\frac{1}{\lambda_n(t_1)}$ is a decreasing function of the argument t_1. Let it be $T_2 > T_1$. In that case, according to formulas (1) and (2) in the previous section, condition.

$$\sum_{i=1}^{n} \alpha_i \xi_i = \sum_{i=1}^{n} \alpha_i \xi_i^j = 1,\ j = 1,2, \ldots \qquad (3)$$

is fulfilled, we have

$$\frac{1}{\lambda(t_j)} = \min_{\xi_1,\ldots,\xi_n} \left(\int_{t_0}^{t_j^j} \sum_{k=1}^{r} |\sum_{k=1}^{r} \xi_i g^k(t)|^{p'} dt \right)^{\frac{1}{p'}} = \left(\int_{t_0}^{t_j^j} \sum_{k=1}^{r} \left|\sum_{k=1}^{r} \xi_i g_i^k(t)\right|^{p'} dt \right)^{\frac{1}{p'}}$$

equalities.

Let`s assume on the contrary $\frac{1}{\lambda_n(t_1^2)} < \frac{1}{\lambda_n(t_1^1)}$. In that case

$$\frac{1}{\lambda_n(t_1^1)} = \left(\int_{t_0}^{t_1^1} \sum_{k=1}^{r} \left|\sum_{k=1}^{n} \xi_i^1 g_i^k(t)\right|^{p'} dt \right)^{\frac{1}{p'}} > \left(\int_{t_0}^{t_1^2} \sum_{k=1}^{r} \left|\sum_{k=1}^{n} \xi_i^2 g_i^k(t)\right|^{p'} dt \right)^{\frac{1}{p'}} =$$

$$\left(\int_{t_0}^{t_1^1} \sum_{k=1}^{r} \left|\sum_{k=1}^{n} \xi_i^1 g_i^k(t)\right|^{p'} dt + \int_{t_0}^{t_1^2} \sum_{k=1}^{r} \left|\sum_{k=1}^{n} \xi_i^2 g_i^k(t)\right|^{p'} dt \right)^{\frac{1}{p'}} \ge$$

$$\left(\int_{t_0}^{t_1^1} \sum_{k=1}^{r} \left|\sum_{k=1}^{n} \xi_i^2 g_i^k(t)\right|^{p'} dt \right)^{\frac{1}{p'}} \qquad (5)$$

But inequality (5) means that if ξ_i^2 is taken instead of $\frac{1}{\lambda_n(t_1^1)}$ in equation (4), then (4) will have a smaller value compared to $\xi_i, i = 1, \ldots, n$. This contradicts our assumption that $\lambda_n(t_1^1)$ This contradicts our assumption that $\lambda_n(t_1^1)$ is the minimum value of expression (4). This contradiction is caused by the hypothesis $\frac{1}{\lambda_n(t_1^2)} < \frac{1}{\lambda_n(t_1^1)}$ that we have accepted . So this assumption is wrong and $\frac{1}{\lambda_n(t_1^2)} \geq \frac{1}{\lambda_n(t_1^1)}$ is correct.

In addition, it is often possible to see the problem of l -moments in the case where the numbers $a_1, \ldots, a_n$ are a function of the value T , that is, $a_1 = a_1(T), \ldots, a_n = a_n(T)$. Then it can be easily seen that λ_n is also a function of T, $\lambda_n = \lambda_n(T)$. But in this case $\lambda_n(T)$ the function cannot be said to have some mooton property. In this case, in the problem of finding the minimum value of $T = T^*$, it is necessary to find the smallest real and non-negative value of T^*that satisfies the inequality (1).

Now we show that the typical problems related to the optimal control of the object expressed by ordinary differential equations can be brought to the problem of moments . . $t(t_0 \leq t \leq t_1)$ and the state of the controlled object at the moment $X_1 = X_1(t), \ldots, X_n = X_n(t)$ is given by the coordinates representing the points. Let the control effect on the moment of time t be expressed by the quantity $u = u_1(t), \ldots, u_r = u_r(t)$. In that case, the movement of the object can be expressed by a system of n -order differential equations with variable:

$$\dot{X}(t) = \frac{dX(t)}{dt} = A(t)X(t) + B(t)u(t) = C(t) \qquad (6)$$

where $X = X(t) = \big(q_1(t)\big), C = C(t) = \big(C_1(t)\big)$ $n-$ order one-column matrix; $u = u(t) = \big(u_k(t)\big)$ $r-$ order one-column matrix; $A = A(t) = \big(a_{ij}(t)\big)$ $n \times n-$ square matrix; $i, j = 1, \ldots, n$; $B = B(t) = \big(b_{ik}(t)\big)$ $n \times r-$ dimensional rectangular matrix; $i = 1, \ldots n$; $k = 1, \ldots r$. (6) the initial state of the system is given by a one-column matrix $X_0 = X_0(t_0) = \big(X_{0i}(t)\big)$. In addition, in system space (6) there is a point moving according to the law represented by a one-column matrix$X^* = X^*(t) = (X_i^*(t))$.

The problem of optimal management is as follows. It is necessary to find the function $u(t)$ $t_0 \leq t \leq t_1$ so that the point X(t) moving by equation (6) moves from the initial state X_0 to the limit state $X(t)$ and the following two conditions are fulfilled:

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(A) Let the norm ‖u‖ of the controlled function $u(t)$ in the space $L_p(t_0, t_1)$ (where $p \geq 1, t_0$ and t_1 are given) reach the minimum value.

(B) $L_p(t_0, t_1)$ the norm of the controlled function in the space is less than the value $l > 0$,

$$\|u\| \leq l \qquad (7)$$

let the control time be t_1 minimal when the condition is met.

Now we will show how it is possible to write problems A and B in the form of moment problem. (6) compatible with the system

$$X = A(t)X \qquad (8)$$

Let the fundamental matrix of the solution of the homogeneous system be $\Phi(t) = \left(\varphi_{ij}(t)\right)$

Then it is known that the function $X(t)$ satisfying the equation (6) can be written as follows when the initial condition $X(t_0) = X_0$ is fulfilled:

$$X(t) = \Phi(t)X_0 + \Phi(t), \Psi(\tau)C(\tau)d\tau + \int_{t_0}^{t} \Psi(\tau)B(\tau)u(\tau)d\tau, \qquad (9)$$

Here $\Phi(t) = \left(\Psi_{ij}(t)\right), i, j = 1, \ldots, n - (8)$ is the fundamental matrix of the solution of the joint system to the system:

$$\dot{\Psi}(t) = A'(t)\Psi(t) \qquad (10)$$

where the matrix $A'(t)$ is the transposed matrix of the matrix $A(t)$.

The matrices $\Phi(t)$ and $\Psi(t)$ have the following properties:

$$\Psi(t) = \Phi^{-1}(t), \qquad (11)$$

$$\Psi(-t) = \Phi(t), \qquad (12)$$

$$\Phi(0) = \Phi(0) = E \qquad (13)$$

where the $E -$matrix is an $n \times n -$ dimensional unit matrix.

(9) we integrate using the boundary condition $X^*(t_1) = X(t_1)$:

$$X^*(t_1) = \Phi(t_1)X_0 + \Phi(t_1)\int_{t_0}^{t_1} \Psi(\tau)C(\tau)d\tau + \Phi(t_1)\int_{t_0}^{t_1} \Psi(\tau)B(\tau)u(\tau)d\tau. \quad (14)$$

By multiplying both sides of this equation by $\Psi(t)$ and using equality (11), the matrices $\Phi(t)$ and $\Psi(t)$ are non-unique matrices for any $t \in [t_0, t_1]$ from uncomplicated transformations then the following equality must be fulfilled for equality (14):

$$\int_{t_0}^{t_1} G(\tau)u(\tau)d\tau = a(t_1) \qquad (15)$$

here

$$a(t_1) = \Psi(t_1)q^*(t_1) - \int_{t_0}^{t} \Psi(\tau)C(\tau)d\tau - X_0, \qquad (16)$$

$$G(t) = \Psi(\tau)B(\tau) \qquad (17)$$

In particular, when the excitation $C(t) = 0$ and the system representing the point are brought to the coordinate origin $X^*(t_1) = 0$ (15), the equation changes to the following form:

$$\int_{t_0}^{t_1} G(\tau)u(\tau)d\tau = -X_0 \qquad (18)$$

Thus, equality (15) or for a special case (18) equality (6) represents the sufficient and necessary conditions satisfying the function $u(t)$ that ensures the transition of the system from the given initial state to the given limit state. On the other hand, these equations represent the problem of moments written in vector-matrix form; where $G(t) = \left(X_i^k(t)\right)$ this $X_i^k(t), i = 1,2, \ldots, n; k = 1,2, \ldots, r$ has $n \times r$ elements is a dimensional matrix.

According to theorem 2 in the previous section, problem A has the following form;

$$u(t) = \lambda_n |\xi G(t)|^{p'-1} sign \xi G(t), t_0 \le t \le t_1, p' \ge 1, \qquad (19)$$

where $\xi = (\xi_1, \ldots, \xi_n)$ is a vector andλ_n is the solution of the following problem:

$a(t_1)\xi = 1$ (20)

when the condition is met

$\min_\xi \left(\int_{t_0}^{t_1} |\xi G(t)|^{p'} dt\right)^{\frac{1}{p'}} = \frac{1}{\lambda_n}$ (21)

must be found. It should be noted here that $\xi G(t)$ term

$\left|\sum_{i=1}^{n} \xi_i g_i^k(t)\right|, \ k = 1,2,\dots,r$ (22)

is a vector with coordinates. The minimum norm of optimal control consists of $\|u\| = \lambda_n$.

To solve problem B, when condition (20) is fulfilled, it is necessary to find the minimum function of (21) $\lambda_n = \lambda_n(t_1)$ .And then

$\lambda_n(t_1^*) \leq l$ (23)

the smallest real non-negative number t_1^* satisfying the inequality is found .

After that, the solution of problem B will be as follows:

$u(t) = l|\xi G(t)|^{p'-1} sign \xi G(t), t_0 \leq t \leq t_1^*,$ (24)

where ξ is the solution for the case $t = t_1^*$ of (21) and (20).

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