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Surjective Quadratic Operator Corresponding to Some Self-Couplings

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Abstract:

Any surjective quadratic operator defined on the simplex S^3 corresponds to some self matching. This operator is a homeomorphism of the simplex S^3. A quadratic operator defined on the simplex S^3 is surjective if and only if it is bijective.

Keywords: surjective, quadratic, operator, simplex, homeomorphism. bijective. self-combination, tetrahedron, transformations, group, vertex, displacement, convex, linear, combination, composition.

On an $S^3 = \left\{ (x_1, x_2, x_3, x_4) : x_l \geq 0,\ l = \overline{1,4};\ \sum_{l=1}^{4} x_l = 1 \right\}$ arbitrary quadratic operator V defined as follows

$$(Vx)_k = \sum_{l,j=1}^{4} P_{lj,k}\, x_l x_j,\ \ k = \overline{1,4} \quad (1)$$

where $P_{l\,j,k} \geq 0,\ P_{l\,j,k} = P_{j\,l,k},\quad \sum_{l,j=1}^{4} P_{l\,j,k} = 1$

We define 24 classes of surjective quadratic operators and prove that they exhaust the entire set of surjective quadratic operators. To describe these classes, we use the well-known self-coincidence groups of regular polyhedra [1], since S^3 is a regular tetrahedron.

Note that self-combination refers to displacement, i.e. metric-preserving transformation. The self-alignment group of the tetrahedron in R^3 consists of 12 elements. But if we consider a simplex in R^4, then it is easy to show that the group of self-combinations of the tetrahedron, G in, consists of the group of all permutations of the vertices of this tetrahedron, i.e. $G = \{\pi_l\}_{l=1}^{24}$.

We say that a quadratic operator V defined on a simplex corresponds to some self-matching if V maps vertices of the simplex to S^3 vertices and edges of the simplex to π_l edges in the same way as self-matching S^3 $\pi_l, l = \overline{1,24}$.

Theorem 1.1. Any surjective quadratic operator defined on the simplex S^3 corresponds to some self matching $\pi_l, l = \overline{1,24}$.

We reduce the proof of Theorem 1.1 to the proof of the following three lemmas.

Lemma 1.1. Let V-surjective quadratic operator. Then no interior point of the simplex S^3 cannot go when mapping V to one of the vertices of the simplex.

Lemma 1.2. Let V be a surjective quadratic operator. Then no interior point of the simplex S^3 can pass under the mapping V to the boundary point of the simplex.

Lemma 1.3. Let V be a surjective quadratic operator. Then no boundary point other than vertices can go under the mapping V to one of the vertices of the simplex.

Proof of Theorem 1.1. By virtue of Lemmas 1.1-1.3, the surjective quadratic operator maps vertices of a simplex to vertices and edges to edges, i.e. a surjective quadratic operator corresponds to some self-combination π_l, $l=\overline{1,24}$.

Let us now determine what kind of quadratic operators correspond to each self-alignment of a regular tetrahedron.

Let's start with identical self-combination π_1. The quadratic operator V corresponding to this self-matching must satisfy the following conditions: $V(A_l)=A_l$, $l=1,2,3,4$ and also

$$V([A_1,A_2])=[A_1,A_2],\ V([A_1,A_3])=[A_1,A_3], V([A_1,A_4])=[A_1,A_4]$$
$$V([A_2,A_3])=[A_2,A_3],\ V([A_2,A_4])=[A_2,A_4], V([A_3,A_4])=[A_3,A_4]$$

If we rewrite these conditions using (1), taking into account that $A_1(1,0,0,0), A_2(0,1,0,0), A_3(0,0,1,0), A_4(0,0,0,1)$ then we get the following relations:

$$\begin{array}{llll} P_{11,1}=1 & P_{22,1}=0 & P_{33,1}=0 & P_{44,1}=0 \\ P_{11,2}=0 & P_{22,2}=1 & P_{33,2}=0 & P_{44,2}=2 \\ P_{11,3}=0 & P_{22,3}=0 & P_{33,3}=1 & P_{44,3}=0 \\ P_{11,4}=0 & P_{22,4}=0 & P_{33,4}=0 & P_{44,4}=1 \end{array} \quad (2)$$

Now, since an arbitrary point belonging to the edge $[A_1,A_2]$ has coordinates $(x_1,1-x_1,0,0)$ then from $V([A_1,A_2])=[A_1,A_2]$ has

$$0=x_3'=P_{11,3}x_1^2+P_{22,3}(1-x_1)^2+2P_{12,3}x_1(1-x_1)$$

$$0=x_4'=P_{11,4}x_1^2+P_{22,4}(1-x_1)^2+2P_{12,4}x_1(1-x_1)$$

And from (1) it follows that $2P_{12,3}=0$, $2P_{12,4}=0$ where $P_{12,3}=0, P_{12,4}=0$; similarly from $V([A_1,A_3])=[A_1,A_3], V([A_1,A_4])=[A_1,A_4]$, $V([A_2,A_3])=[A_2,A_3]$, $V([A_2,A_4])=[A_2,A_4]$, $V([A_3,A_4])=[A_3,A_4]$.

We have

$$P_{23,1}=0\quad P_{24,1}=0\quad P_{34,1}=0\quad P_{13,2}=0\quad P_{14,2}=0$$

$$P_{14,3}=0\quad P_{24,3}=0\quad P_{34,2}=0\quad P_{13,4}=0\quad P_{23,2}=0$$

Thus, the quadratic operators corresponding to self-matching π_1 have the following form:

$$V_1(\alpha,\beta,\gamma,\xi,\eta,\delta)=\begin{bmatrix} 1 & 0 & 0 & 0 & \alpha & \beta & \gamma & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1-\alpha & 0 & 0 & \xi & \eta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1-\beta & 0 & 1-\xi & 0 & \delta \\ 0 & 0 & 0 & 1 & 0 & 0 & 1-\gamma & 0 & 1-\eta & 1-\delta \end{bmatrix}$$

Where $\alpha,\beta,\gamma,\xi,\eta,\delta \in [0,1]$ - arbitrary numbers.

Obviously, a convex linear combination of quadratic operators corresponding to the self-matching π_1 also corresponds to this self-matching.

Let us show that the quadratic operator $V_1(1/2,\ 1/2, 1/2, 1/2, 1/2)$ coincides with self-combination indeed, at $\alpha,\beta,\gamma,\xi,\eta,\delta = 1/2$ квадратичный оператор $V_1(1/2,\ 1/2, 1/2, 1/2, 1/2, 1/2)$ is the identity operator, because.

$$\begin{cases} x_1' = x_1\left(x_1 + x_2 + x_3 + x_4\right) \\ x_2' = x_2\left(x_1 + x_2 + x_3 + x_4\right) \\ x_3' = x_3\left(x_1 + x_2 + x_3 x_4\right) \\ x_4' = x_4\left(x_1 + x_2 + x_3 + x_4\right) \end{cases}$$

whence due to the fact that $x_1 + x_2 + x_3 + x_4 = 1$, we get that $V_1(1/2,\ 1/2, 1/2, 1/2, 1/2, 1/2)$ coincides with self-combination π_1.

For quadratic class operators $V_1(\alpha,\beta,\gamma,\xi,\eta,\delta)$ transformation (1) takes the form:

$$\begin{cases} x_1' = x_1[1+(2\alpha-1)x_2 + (2\beta-1)x_3 + (2\gamma-1)x_4] \\ x_2' = x_2[1+(1-2\alpha)x_1 + (2\xi-1)x_3 + (2\eta-1)x_4] \\ x_3' = x_3[1+(1-2\beta)x_1 + (1-2\xi)x_2 + (2\delta-1)x_4] \\ x_4' = x_4[1+(1-2\gamma)x_1 + (1-2\eta)x_2 + (1-2\delta)x_3] \end{cases} \quad (3)$$

A quadratic operator of the form [3] belongs to the class of Voltaire operators. This class of operators was considered in [3]. In particular, for Volterian quadratic operators it was proved that operators of this type are one-to-one and mutually continuous operators [3]. Hence we have the following.

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